

Propagation dynamics of an optical vortex imposed on an Airy beam

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In this Letter, we demonstrate the general propagation dynamics of an Airy beam (AiB) carrying unit phase singularity, i.e., optical vortices (OVs). For the OV with a unit charge, theoretical analysis indicates that the OV carried by the AiB will propagate along the parabolic trajectory with an acceleration velocity twice as fast as conventional AiBs before a critical position. Thereafter, the AiB main lobe destroyed by OV will be reconstructed and the phase singularity will reappear in the middle of the AiB profile. © 2010 Optical Society of America

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In the framework of diffraction-free wave packages, the Airy package is unique for its acceleration property [1]. The original work associated with the Airy package dates back to 30 years ago in the context of solving the Schrödinger equation in quantum mechanics. Recently, its counterpart in optics—a finite energy Airy beam (AiB)—has attracted intense interest due to its nondiffracting and peculiar acceleration properties [2–10]. Most research efforts have been focused on three aspects of AiBs: generation [11,12], propagation dynamics [13], and applications [14–17]. So far, finite energy AiBs have been extended from cw to pulse [18], the spatiotemporal Airy–Gaussian bullet [19], and surface plasmons [20].

The propagating properties of optical vortices (OVs) incorporated in various optical elements, such as Fresnel zone plates [21] and the devil's lens [22], have been studied extensively. Nevertheless, most of the research works are concerned with axial symmetry elements or the ambient wave, for example Gaussian beams [23]. Therefore, a question arises: how does a conventional OV propagate when it is imposed on an AiB with an off-axis situation? Mazilu *et al.* [24] have introduced the OV in the cubic phase profile to study the propagation of the OV with the AiB; however, the propagation dynamics cannot be clearly apprehended by the model. Meanwhile, from a practical point of view, superimposing an OV on an AiB to generate a doughnut main lobe in AiB for simultaneously trapping and cleaning [15] microparticles is of interest. In this Letter, we directly superimpose the spiral phase on the AiB, by which the propagation of a single-charge OV carried by the AiB can be characterized explicitly. Theoretical results show that, similarly to the propagation of AiBs, the OV will also have a transverse shift but with much faster acceleration velocity. As a result, the OV, though delayed from the AiB main lobe at the initial position, will overlap with the main lobe after a propagation distance, thus generating a doughnut profile.

The initial finite energy AiB superimposed by a spiral phase in a Cartesian coordinate can be expressed as [2]

$$u(x, y, z = 0) = \text{Ai}(x/x_0)\text{Ai}(y/y_0) \exp[a(x/x_0 + y/y_0)] \\ \times [(x - x_d) + i(y - y_d)]^l,$$

where x_0 , y_0 , and a are the beam parameters with the same meaning ascribed in [2]; x_d and y_d denote the dislocation of the OV from the origin along the x and y axes, respectively; and l represents the topological charge of the OV. The center of the OV (COV) is defined as the symmetric center of the phase profile, whereas the origin ($x = 0$ and $y = 0$) of the coordinate is set as the initial position of the AiB main lobe. The evolution of the field can be calculated through the Fresnel diffraction integral [13]:

$$u(x, y, z) = \frac{ik}{2\pi z} \iint \exp\left(-i\frac{k}{2z}((x_1 - x)^2 + (y_1 - y)^2)\right) \\ \times u(x_1, y_1, z = 0) dx_1 dy_1 = \text{FreT}\{u(x, y, z)\}. \quad (1)$$

For simplicity, here we choose a unit topological charge; hereafter, all phase singularities or OVs are referred to as this special topological charge value. Therefore, Eq. (1) can be simplified as

$$u(x, y, z) = \text{FreT}\{\text{Ai}(x/x_0)\text{Ai}(y/y_0) \exp[a(x/x_0 + y/y_0)] \\ \times [(x - x_d) \pm i(y - y_d)]\}. \quad (2)$$

The integral in Eq. (2) can be derived from Eq. (9) in [13]. Finally, the analytical complex field of the AiB carrying unit spiral phase after propagating a distance z can be formulated as

$$u(x, y, z) = \frac{1}{kx_0y_0} \exp(p(x, y, z))(I_1 + I_2 + I_3), \quad (3)$$

where

$$p(x, y, z) = a\left(\frac{x}{x_0} + \frac{y}{y_0}\right) - 2a\left(\frac{x_m(z)}{x_0} + \frac{y_m(z)}{y_0}\right) \\ + i\left[\frac{z}{2k}\left(\frac{x}{x_0^3} + \frac{y}{y_0^3}\right) + \frac{a^2z}{2k}\left(\frac{1}{x_0^2} + \frac{1}{y_0^2}\right)\right. \\ \left. - \frac{z^3}{12k^3}\left(\frac{1}{x_0^6} + \frac{1}{y_0^6}\right)\right], \quad (4)$$

with $k = 2\pi/\lambda$, λ is the wavelength of the incident light, and

$$\begin{aligned} I_1 &= kx_0y_0\text{Ai}(q(x))\text{Ai}(q(y))((x - x_d - 2x_m(z)) \\ &\quad \pm i(y - y_d - 2y_m(z))), \end{aligned} \quad (5)$$

$$I_2 = \mp x_0z\text{Ai}(q(x))(A\text{i}'(q(y)) + a\text{Ai}(q(y))), \quad (6)$$

$$I_3 = iy_0z\text{Ai}(q(y))(\text{Ai}'(q(x)) + a\text{Ai}(q(x))), \quad (7)$$

with $\text{Ai}'(\cdot)$ representing the derivative of the Airy function and $q(s) = \frac{(s-s_m(z))}{s_0} + \frac{iaz}{ks_0^2}$, $s_m(z) = \frac{z^2}{4k^2s_0^3}$, $s = x$ or y .

From Eqs. (5)–(7), we see that I_1 represents a conventional AiB imposed with an OV, whose center is located at point $(x_d + 2x_m(z), y_d + 2y_m(z))$. I_2 depicts the beam profile with the x component being described by the Airy function, while the y component is the combination of the Airy function and its derivative. I_3 is similar to I_2 but with the x and y coordinates exchanged and an additional advanced phase of $\pi/2$ due to the imaginary unit.

Figure 1(a) shows the field distribution of an AiB with phase singularity after a propagation distance ($z = 34.38$ cm); the beam parameters are set as $a = 0.05$, $x_0 = 100 \mu\text{m}$, and $y_0 = 100 \mu\text{m}$, and the dislocation is $x_d = y_d = 300 \mu\text{m}$. The wavelength of incident light used for calculation is 633 nm from a He–Ne laser. Hereafter, the parameters used are the same as those in Fig. 1 unless otherwise stated. The observed doughnut main lobe in Fig. 1(a) is attributed to the superposition between the OV and AiB. Figures 1(b)–1(d) show three decomposed intensity profiles of Fig. 1(a), which correspond to I_1 , I_2 , and I_3 , respectively. It is obvious that I_1 describes the AiBs carried with OVs, while I_2 and I_3 present only a half

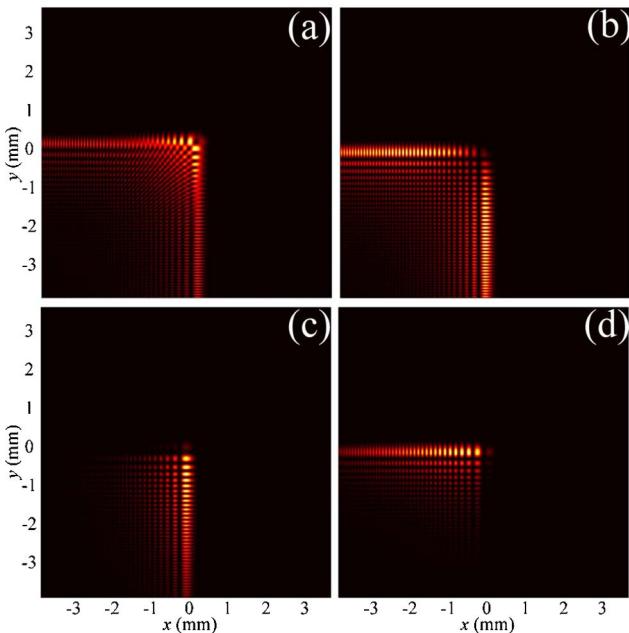


Fig. 1. (Color online) Intensity profile of the OV carried by the AiBs located at (a) position $z = 34.38$ cm and the three individual components (b) I_1 , (c) I_2 , and (d) I_3 . The beam parameters are all set as $a = 0.05$, $x_0 = y_0 = 100 \mu\text{m}$, and $x_d = y_d = 300 \mu\text{m}$ with a 633 nm laser illuminating.

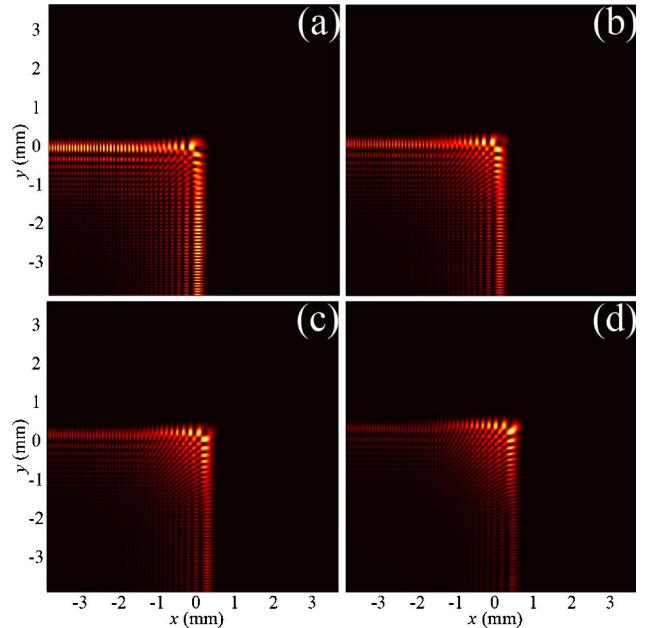


Fig. 2. (Color online) Superposition between the OV and the AiB main lobe at the location close to critical distance z_s , (a) $z = 0.7 z_s$, (b) $z = 0.9 z_s$, (c) $z = 1.1 z_s$, and (d) $z = 1.3 z_s$.

side of the Airy profile. However, it is worth mentioning that the asymmetric intensity distribution of I_2 [Fig. 1(c)] and I_3 [Fig. 1(d)] due to the phase delay yields the non-uniform doughnut profile, as shown in Fig. 1(a). According to theoretical and numerical analysis, the location of the OV can be determined from term I_1 , while other terms (including interference terms) in $|u(x, y, z)|^2$ have little effect.

As aforementioned, it is obvious from part I_1 that the acceleration velocity [$v = 2 \frac{d}{dz} \sqrt{x_m^2(z) + y_m^2(z)}$] of the OVs is twice that of the main lobe of the conventional AiB. At position $z_s = 2 kx_0 \sqrt{x_0 x_d}$ (only for $x_0 = y_0$), the spiral phase will superimpose completely on the main lobe of the AiB; i.e., the deflection of the OV is equal to that of the AiB, yielding a doughnut main lobe. In Fig. 1, the propagation distance used is just the critical value, and the doughnut profile is well formed in Fig. 1(a) as the theoretical anticipation. Nevertheless, the OV cannot exceed the main lobe of the AiB because of the null energy distribution of the AiB governed by the Airy function. The combination of I_2 and I_3 will destroy the regular profile of the AiB because of the invert phase between the Airy function and its derivative. The evolution of the superimposition of the OV with the AiB main lobe is illustrated in Figs. 2(a)–2(d), which plot the intensity cross section of the AiB with the OV at various propagation distances (range from $0.7 z_s$ to $1.3 z_s$).

From Eq. (3), the dislocation between the COV and the origin has almost no effect upon the propagation of the AiB with the OV, except at the position where the OV and main lobe of the AiB superimpose each other. Similarly to a conventional AiB, the beam parameters x_0 and y_0 will dominate the acceleration velocity [3] as well as the critical propagation distance z_s . The finite energy parameter a appears in both the exponential factor and I_2 and I_3 , which contributes to not only the apodized aperture

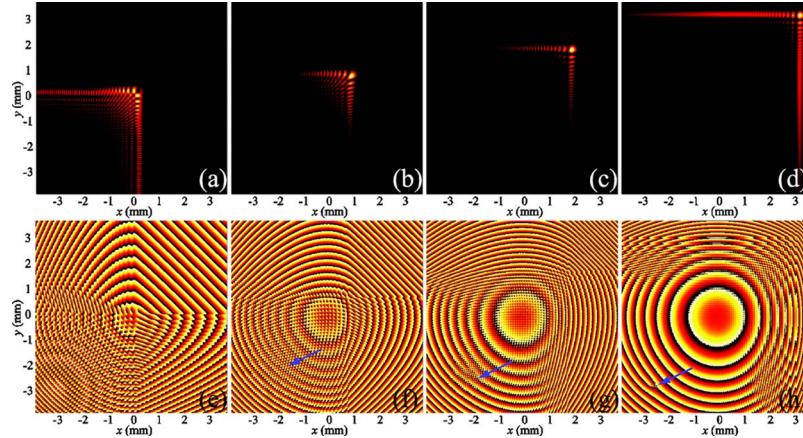


Fig. 3. (Color online) Propagation of AiBs with OV after critical distance z_s : (a)–(d) show the intensity distribution located at $1 z_s$, $1.7 z_s$, $2.3 z_s$, and $3 z_s$, respectively; (e)–(h) show the corresponding phase distribution—the phase singularity positions have been highlighted by blue arrows.

but also the distortion of the two wings, as shown in Fig. 1(a).

As the beam propagates further, the main lobe is reconstructed due to the AiB's intrinsic nondiffracting property (self-healing). Figures 3(a)–3(d) describe the propagation of the AiBs carrying the OV after the critical position z_s , and Figs. 3(e)–3(h) show the corresponding phase distribution. A clear forklike pattern can be observed in Figs. 3(g) and 3(h), which proves the regeneration of the OV. The revival main lobe will continue to propagate along the parabolic trajectory with a slight deviation from the conventional AiBs, as shown in Fig. 4, which is due to the two distorted branches as aforementioned. For more complicated cases with larger or fractional topological charges, the theoretical analysis and numerical simulation employed here can be applied as well. The results will be published elsewhere.

In conclusion, we have simulated that the propagation dynamics of an AiB superimposed with a unit topological charge OV. Theoretical analysis indicates that the OVs will propagate along a parabolic trajectory, which is similar to a conventional AiB but with twice the acceleration velocity. Nevertheless, its transverse shift distance cannot exceed a critical position, where the OV will

superimpose on the main lobe of the AiB fully. After the critical position, the phase singularity will gradually reappear within the Airy pattern.

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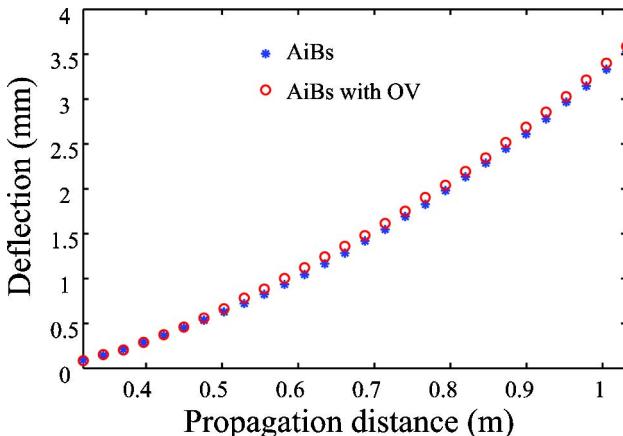


Fig. 4. (Color online) Trajectory of the main lobe peak of AiBs with (red open circles) and without (blue stars) OV after the critical position z_s .